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CLOSED FORM SOLUTIONS OF PROBLEMS ON THE ELASTIC EQUILIBRIUM OF AN INFINITE WEDGE WITH NONSYMMETRIC NOTCH AT THE APEX

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The solution is given of a number of problems on the elastic equilibrium of an infinite wedge with a nonsymmetric notch at the apex by using the method elucidated in [1]. The solution is obtained in the form of the Cauchy-type integrals for various homogeneous conditions on the side faces of the wedge.

The problem of representing 2×2 matrices given on a curve L in the complex plane and belonging to a certain class is posed and solved in closed form in [2, 3] in the form of the product of 2×2 matrices holomorphic to the left and right of L, whose boundary conditions on L commutate.

A simpler solution of the mentioned homogeneous Hilbert problem, more convenient for applications, is given in [1]. It is also shown here that the problem of elastic equilibrium of an infinite wedge with nonsymmetric notch at the apex and stress-free faces reduces to an inhomogeneous Hilbert problem for a two-dimensional piecewise-holomorphic vector, where the matrix factor belongs to the above-mentioned class in three cases.

1. Reduction of the problem of elastic equilibrium of a wedge with a notch to an inhomogeneous Hilbert problem. Let an infinite triangular wedge occupy the domain $0 \leqslant \varphi \leqslant \theta$ in a plane with the polar coordinates r, φ. Values of the stresses σ_φ, $\tau_{r\varphi}$ are given on the face $\varphi = 0$, but on the face $\varphi = \theta$ we consider homogeneous conditions of one of the following kinds :

$$(1)\quad \sigma_\varphi = \tau_{r\varphi} = 0, \qquad (3)\quad u_\varphi = \tau_{r\varphi} = 0$$
$$(2)\quad u_r = u_\varphi = 0, \qquad (4)\quad u_r = \sigma_\varphi = 0$$

to be satisfied.

Let us introduce the following two-dimensional vectors and their Mellin transforms:

$$\sigma(r) = \{\sigma_\varphi(r,0);\ \tau_{r\varphi}(r,0)\}, \qquad \sigma^\circ(p) = \int_0^\infty r^p \sigma(r)\,dr \tag{1.1}$$

$$w(r) = \left\{\frac{\partial u_\varphi}{\partial r}(r,0);\ \frac{\partial u_r}{\partial r}(r,0)\right\}, \qquad w^\circ(p) = \int_1^\infty r^p w(r)\,dr \tag{1.2}$$

For conditions (1) - (4), we obtain relationships between the vector-transforms ($j = 1-4$, respectively)

$$^1/_4 E w^\circ(p) = [C + \Delta_j^{-1} G_j(p,\theta)]\,\sigma^\circ(p) \tag{1.3}$$

$$C = \begin{Vmatrix} 0 & -{}^1/_4(1-\nu) \\ {}^1/_4(1-\nu) & 0 \end{Vmatrix} \qquad 2G_j(p,\theta) = \begin{Vmatrix} a_j^+ & b_j^+ \\ b_j^- & a_j^- \end{Vmatrix}$$

$$\Delta_1(p,\theta) = p^2 \sin^2\theta - \sin^2 p\theta$$
$$a_1^\pm = \pm p \sin\theta \cos\theta + \sin p\theta \cos p\theta, \qquad b_1^\pm = -p(1 \mp p)\sin^2\theta$$
$$\Delta_2(p,\theta) = p^2 \sin^2\theta + \varkappa \sin^2 p\theta - {}^1/_4(\varkappa+1)^2$$
$$a_2^\pm = \pm p \sin\theta\cos\theta - \varkappa \sin p\theta \cos p\theta$$
$$b^\pm_2 = -p(p \mp 1)\sin^2\theta - {}^1/_4(\varkappa^2 - 1)$$
$$\Delta_3(p,\theta) = p \sin 2\theta + \sin 2p\theta$$
$$a_3^\pm = \pm \cos 2\theta - \cos 2p\theta, \qquad b_2^\pm = (-1 \pm p)\sin 2\theta$$
$$\Delta_4(p,\theta) = p\sin 2\theta - \sin 2p\theta$$
$$a_4^\pm = \pm \cos 2\theta + \cos 2p\theta, \qquad b_4^\pm = (-1 \pm p)\sin 2\theta$$

Here E and ν are the elastic modulus and Poisson's ratio, $\varkappa$ is the coefficient in the Kolosov-Muskhelishvili formulas, and only the upper or lower signs are taken in the expressions for $a_j^\pm$ and $b_j^\pm$.

Now, let there be a notch (0,1) on the $\varphi = 0$ line of the wedge $-\theta_2 \leqslant \varphi \leqslant \theta$, on whose boundaries equal and opposite stresses are applied. Conserving the previous notation $\sigma(r)$ and $\sigma^\circ(p)$ for the original and the transform of the two-dimensional stress vector (1.1) on the notch line $\varphi = 0$, let us introduce still another two-dimensional vector of the mutual displacement of the edges $v(r)$ and its derivative $u(r)$ by means of the formulas

$$v(r) = \{u_\varphi(r,+0) - u_\varphi(r,-0),\ u_r(r,+0) - u_r(r,-0)\} \tag{1.4}$$
$$u(r) = v'(r) = w(r,+0) - w(r,-0) \tag{1.5}$$

The corresponding Mellin transforms are

$$v^\circ(p) = \int_0^\infty r^p v(r)\,dr = \int_0^1 r^p v(r)\,dr \tag{1.6}$$

$$u^{\circ}(p) = \int_0^{\infty} r^p u(r)\, dr = \int_0^{1} r^p u(r)\, dr \tag{1.7}$$

If the j th of the conditions (1) - (4) is satisfied on the face $\varphi = \theta_1$ and the k th on the face $\varphi = -\theta_2$, we obtain by virtue of (1.3) and (1.5):

$$^1/_4 E u^{\circ}(p) = G_{jk}(p; \theta_1, \theta_2)\, \sigma^{\circ}(p) \tag{1.8}$$

$$G_{jk}(p; \theta_1, \theta_2) \doteq \Delta_j^{-1}(p, \theta_1)\, G_j(p, \theta_1) - \Delta_k^{-1}(p, -\theta_2)\, G_k(p, -\theta_2) \tag{1.9}$$

Let us represent the vector Mellin transform $\sigma^{\circ}(p)$ as

$$\sigma^{\circ}(p) = \sigma^{\circ +}(p) + \sigma^{\circ -}(p), \quad \sigma^{\circ +}(p) = \int_0^1 r^p \sigma(r)\, dr, \quad \sigma^{\circ -}(p) = \int_1^{\infty} r^p \sigma(r)\, dr \tag{1.10}$$

Here $\sigma^{\circ +}(p)$ is holomorphic in the half-plane $\operatorname{Re} p > -{}^1/_4$ and $\sigma^{\circ -}(p)$ in the half-plane $\operatorname{Re} p < 0$, where $\sigma^{\circ +}(p)$ is given. By virtue of (1.7), the transform $u^{\circ}(p)$ is holomorphic in the half-plane $\operatorname{Re} p > -{}^1/_4$. Hence, an expression such as

$$\varphi(p) = \begin{cases} {}^1/_4 E u^{\circ}(p), & \operatorname{Re} p > \delta_0 \\ \sigma^{\circ -}(p), & \operatorname{Re} p < \delta_0 \end{cases} \tag{1.11}$$

is a piecewise-holomorphic two-dimensional vector with the line of jumps $\operatorname{Re} p = \delta_0$, where δ_0 is an arbitrary real number from the interval $(-{}^1/_4, 0)$.

By virtue of (1.8) the relationship

$$\varphi^+(t) = G_{jk}(t; \theta_1, \theta_2)\, \varphi^-(t) + \psi(t) \qquad \psi(t) = G_{jk}(t; \theta_1, \theta_2)\, \sigma^{\circ +}(t) \tag{1.12}$$

is satisfied on the contour defined by the equation $\operatorname{Re} p = \delta_0$. Here $\varphi^{\pm}(t)$ denote the limit values of the piecewise-holomorphic vector $\varphi(p)$ on L from the left and right, respectively.

Since $\psi(t)$ is a known vector, the relationship (1.12) is an inhomogeneous Hilbert problem, which is solved in closed form if the solution of the corresponding homogeneous problem is known

$$X^+(t)\, [X^-(t)]^{-1} = G_{jk}(t; \theta_1, \theta_2) \tag{1.13}$$

where $X(p)$ is a piecewise-holomorphic matrix with the line of jumps L.

2. Schemes of the work of a wedge in which the matrices commutate. As is known [1 -3], the solution of the problem (1.13) satisfies the additional condition

$$X^+(t)\, [X^-(t)]^{-1} = [X^-(t)]^{-1} X^+(t)$$

in one case, and admits of solution in the form of Cauchy-type integrals.

In order for the case under consideration to hold, it is necessary to satisfy the following condition [1]:

$$\operatorname{dev} G_{jk}(t; \theta_1, \theta_2) = c(t; \theta_1, \theta_2)\; Q_{jk}(t; \theta_1, \theta_2) \tag{2.1}$$

$$\operatorname{dev} G_{jk}(t; \theta_1, \theta_2) = G_{jk}(t; \theta_1, \theta_2) - b(t; \theta_1, \theta_2)\, I$$

Here $b(t; \theta_1, \theta_2)$ is semi-trace of the matrix $G_{jk}(t; \theta_1, \theta_2)$, I is the unit matrix 1×2, $c(t; \theta_1, \theta_2)$ is an arbitrary coefficient, and $Q_{jk}(t; \theta_1, \theta_2)$ is the boundary value on L of a 2×2 matrix of the form

$$Q_{jk}(p;\theta_1,\theta_2) = \begin{Vmatrix} l(p) & m^+(p) \\ m^-(p) & -l(p) \end{Vmatrix} \qquad (2.2)$$

where $l(p)$, $m^+(p)$, $m^-(p)$ are polynomials in p.

If the number of zeroes of odd multiplicity of the polynomial $(l^2 + m^+m^-)(p)$, each taken once, does not exceed two, then (2.1) is also a sufficient condition; otherwise additional analysis is required [1].

From (1.9) we have directly

$$\operatorname{dev} G_{jk}(p;\theta_1,\theta_2) = q_j(p,\theta_1)\,Q_j(p,\theta_1) - q_k(p,-\theta_2)\,Q_k(p,-\theta_2) \qquad (2.3)$$

$$2Q_j(p,\theta) = \begin{Vmatrix} l_j & m_j^+ \\ m_j^- & -l_j \end{Vmatrix}$$

$$q_1(p,\theta) = \Delta_1^{-1}(p,\theta)\,p\sin\theta, \quad l_1 = \cos\theta, \quad m_1^{\pm} = (-1 \pm p)\sin\theta \qquad (2.4)$$

$$q_2(p,\theta) = \Delta_2^{-1}(p,\theta), \quad l_2 = p\sin\theta\cos\theta, \quad m_2^{\pm} = p(\pm 1 - p)\sin^2\theta - {}^1/_4(\varkappa^2 - 1) \qquad (2.5)$$

$$q_3(p,\theta) = \Delta_3^{-1}(p,\theta), \quad l_3 = \cos 2\theta, \quad m_3^{\pm} = (\pm p - 1)\sin 2\theta \qquad (2.6)$$

$$q_4(p,\theta) = \Delta_4^{-1}(p,\theta), \quad l_4 = \cos 2\theta, \quad m_4^{\pm} = (\pm p - 1)\sin 2\theta \qquad (2.7)$$

Here only the upper or lower signs are taken in the formulas for m_j. The matrices $Q_j(p,\theta_1)$ and $Q_k(p,-\theta_2)$ in the right side of (2.3) have polynomial elements, where these elements do not simultaneously all vanish identically with respect to p for any values of the parameters θ_1, θ_2.

If the identity

$$q_k(p,-\theta_2) \equiv 0 \qquad (2.8)$$

is staisfied, then setting

$$c(t;\theta_1,\theta_2) = q_j(t;\theta_1,\theta_2), \quad Q_{jk}(p;\theta_1,\theta_2) = Q_j(p,\theta_1) \qquad (2.9)$$

we obtain (2.1), (2.2) from (2.3).

Analogously to the above, if the identity

$$Q_k(p,-\theta_2) = Q_j(p,\theta_1)\cdot h(p;\theta_1,\theta_2) \qquad (2.10)$$

is satisfied, where $h(p;\ \theta_1,\ \theta_2)$ is an arbitrary scalar factor, then assuming

$$c(t;\theta_1,\theta_2) = q_j(t,\theta_1) - h(t;\theta_1,\theta_2)\,q_k(t,-\theta_2), \quad Q_{jk}(p;\theta_1,\theta_2) = Q_j(p,\theta_1) \qquad (2.11)$$

we again obtain (2.1), (2.2) from (2.3).

There results directly from (2.4) - (2.7) that condition (2.8) is satisfied only when $k = 1$, $\theta_2 = \pi$, i.e., in the case of a notch on the continuation of the stress-free side of the half-plane enveloping the wedge under arbitrary boundary conditions on its opposite side (diagrams *1*—*4* in Fig. 1; the notations (*1*)—(*4*) of the boundary conditions mentioned at the beginning of Sect. 1 are given in the right lower portion of the figure).

Now, let us study in which cases the identity (2.10) is satisfied. Henceforth $h = h(p;\ \theta_1,\ \theta_2)$ throughout.

Initially let us put $j = 1$, releasing the side $\varphi = \theta_1$ from stresses

$$\theta_1 + \theta_2 = \pi,\ h = -1; \ \theta_1 + \theta_2 = 2\pi,\ h = 1 \quad \text{for} \quad k = 1 \qquad (2.12)$$

It hence follows that the identity (2.10) is satisfied for a notch on the boundary of the half-plane, and for a semi-infinite notch with a breakpoint (diagrams *5*,*6*). These diagrams, exactly as the case $j = k = 1$, $\theta_2 = \pi$ (diagram *I*), have been examined in [1]. Let us note that since the diagonal elements of the matrices $Q_1(p, \theta_1)$ and

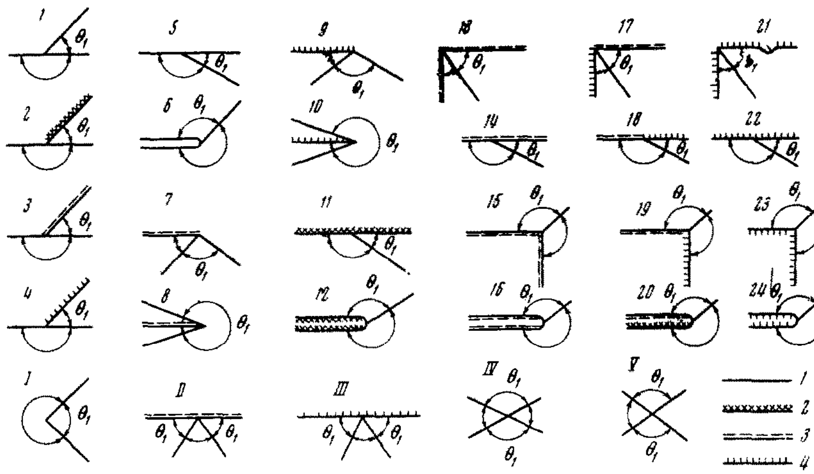

Fig. 1.

$Q_1(p, -\theta_2)$ are constants, the factor h cannot depend on p. Equating elements of the first rows of the matrices $Q_1(p, -\theta_2)$ and $hQ_1(p, \theta_1)$, we easily see that $h = \pm 1$, and this result is valid for all the diagrams considered below.

If $k = 2$, the identity (2.10) is not satisfied for any values of the angles θ_1 and θ_2

If $k = 3$ (diagrams *7*,*8*) or $k = 4$ (diagrams *9*,*10*), we have

$$\theta_1 + 2\theta_2 = \pi, \quad h = -1; \quad \theta_1 + 2\theta_2 = 2\pi, \quad h = 1 \tag{2.13}$$

Now, putting $j = 2$, and fixing the side $\varphi = \theta_1$ rigidly, we have

$$\text{a) } \theta_1 + \theta_2 = \pi, \ h = 1; \ \text{(b) } \theta_1 + \theta_2 = 2\pi, \ h = 1 \quad \text{for } k=2 \tag{2.14}$$

Condition (2.14a) (diagram *11*) corresponds to a notch on the rigidly fixed boundary of the half-plane.

Condition (2.14b) (diagram *12*) defines an arbitrarily oriented notch at the end of an intinitely rigid soldered plate.

If $k = 1, 3, 4$, the identity (2.10) is not satisfied for any value of the angles θ_1 and θ_2.

Assuming contact with an absolutely rigid smooth profile at the side $\varphi = \theta_1$ ($j = 3$), we obtain relationships between the angle θ_1 and θ_2 which assure compliance with the identity (2.10) and are different from those examined earlier, and we have

$$\text{a) } \theta_1 + \theta_2 = {}^1/_2\pi, \ h = -1; \quad \text{b) } \theta_1 + \theta_2 = \pi, \ h = 1 \tag{2.15}$$

$$\text{c) } \theta_1 + \theta_2 = {}^3/_2\pi, \ h = -1; \quad \text{d) } \theta_1 + \theta_2 = 2\pi, \ h = 1 \quad \text{for } k=3$$

In the first case (diagram *13*) we have a notch in a rectangular wedge, compressed by two rigid stamps without friction, in the second case (diagram *14*) we have a notch at the boundary of the half-plane in the presence of a rigid stamp without friction, in the third case (diagram *15*) we have a notch at the point of a rectangular rigid stamp without friction, and in the fourth case (diagram *16*) a notch at the end of an infinitely thin and absolutely rigid smooth "knife-blade ". In all cases it is assumed that the size of the stamp or the knife considerably exceeds the length of the notch.

If $k = 4$, then the relationships (2.15 a - d) again hold, and the corresponding diagrams *17-20* are characterized by the fact that contact with a rigid smooth profile is realized on one of the wedge sides, while contact with a body deprived of bending stiffness and possessing infinite tensile stiffness is realized on the other.

Finally, putting $j = k = 4$, we again obtain the relationships (2.15 a - d). The corresponding diagrams *21-24* are different in that contact conditions with a body deprived of bending stiffness and possessing infinite tensile stiffness are realized on both sides of the wedge.

Such are the fundamental diagrams, where (2.1), (2.2) are satisfied on the sides of a wedge under the homogeneous boundary conditions (1) - (4).

Let us enumerate several derived diagrams.

The state of stress of a body with a pair of converging notches of identical length (diagram *I*) can be obtained by adding the states of stress for diagrams *14* and *22*.

The state of stress of a half-plane with a pair of symmetrically disposed notches and the boundary conditions (3), (4) on the surface (diagrams *II—III*) can be obtained by superposing the diagrams *13, 17, 21*

The problem of the state of stress of a body with a pair of notches of identical length dividing each other in half (diagram *IV*) also reduces to synthesis of diagrams *13, 17, 21*.

Finally, the problem of the state of stress of a half-plane enveloping a wedge with the wedge faces free of stress resultants and a pair of notches of identical length on the continuation of the faces (diagram *V*) reduces to the synthesis of diagrams *7, 9*

Let us note that a diagram of the work of a body with notches inverted relative to the origin, i.e., semi-infinite (one or more), can be compared to each of the fundamental and derived diagrams considered above. The matrix equation (1.13) and the procedure for its solution are completely identical for the original and composite diagrams.

3. General solution of the problem for the fundamental diagrams. Letting $\lambda_{1,2}(p)$ denote the characteristic functions (eigennumbers) of the matrix $G_{jk}(p;\ \theta_1,\ \theta_2)$, let us introduce the following parameters of the given matrix:

$$\Delta(p) = \lambda_1(p)\,\lambda_2(p);\quad \varepsilon(p) = {}^1/_2 \ln[\lambda_1(p)\,\lambda_2^{-1}(p)] \tag{3.1}$$

$$B(p) = [f(p)]^{-1/2}\, Q_{jk}(p;\ \theta_1,\ \theta_2),\ f(p) = [l(p)]^2 + m^+(p)\, m^-(p)$$

Here $\Delta(p)$, $\varepsilon(p)$ and $B(p)$ are the determinant, exponent and commutant of the matrix $G_{jk}(p;\ \theta_1,\ \theta_2)$ [1].

Since the polynomial $f(p)$ has no multiple roots, the solution of the problem (1.13) is given by the following formulas [1]:

$$\Lambda(p)\,\mathrm{X}(p) = I\,\mathrm{ch}\,[(f^{1/2}\beta)(p)] + B(p)\,\mathrm{sh}\,[(f^{1/2}\beta)(p)]$$

$$\Lambda(p) = (p-a)^{\varkappa_\Delta} \exp\left[-\frac{1}{4\pi i}\int\limits_L \frac{\ln\Delta(t)}{t-p}\,dt\right]$$

$$\beta(p) = f^{-1/2}(a)\,\varkappa_\varepsilon \ln(p-a) - \frac{1}{2\pi i}\int\limits_L \frac{f^{-1/2}(t)\,\varepsilon(t)}{t-p}\,dt$$

$$\varkappa_\Delta = (4\pi i)^{-1}[\ln\lambda_1\lambda_2]\,|_L,\qquad \varkappa_\varepsilon = (4\pi i)^{-1}[\ln\lambda_1\lambda_2^{-1}]\,|_L \tag{3.2}$$

where a is an arbitrary point on L.

It is verified directly that the functions $\ln \Delta(t)$ and $\varepsilon(t)$ satisfy the Hölder condition in the neighborhood of infinity, including at the point itself. Since $\varkappa_\varepsilon = 0$, the additional analysis which we mentioned above is to verify compliance with the relationships [1]:

$$\int_L t^{\alpha-1} [f(t)]^{-1/2} \varepsilon(t)\, dt = 0 \qquad (\alpha = 1, \ldots, \gamma) \tag{3.3}$$

Here γ is the greatest of the integers such that the quantity $2\gamma + 1$ does not exceed the degree of the polynomial $f(p)$.

In all the diagrams considered, except diagrams *2, 11, 12*, the degree of the polynomial $f(p)$ equals two, and compliance with relationships of the form (3.8) is not required. For diagrams *2, 11, 12* the corresponding degree is four, and compliance is required with the single relationship

$$\int_L [f(t)]^{-1/2} \varepsilon(t)\, dt = 0 \tag{3.4}$$

The integrand in (3.4) has no singularities in the strip $-1/4 < \operatorname{Re} p \leqslant 0$, where the contour L is located, and tends to zero with infinite growth in the absolute value of p in this strip. The equality (3.4) is verified directly on the imaginary axis, and hence it is valid for any contour L in the strip mentioned.

Letting the quantity δ_0 tend to zero just as has been done in [1], we obtain the following formulas for all the diagrams except *2, 11, 12*:

$$\Lambda(p) \exp\left[-\frac{p}{2\pi}\int_0^\infty \frac{\ln|\Delta(i\tau)|}{\tau^2 + p^2}\, d\tau\right] = \begin{cases} p^{1/2}, & -1/2\pi \leqslant \arg p \leqslant 1/2\pi, \quad \operatorname{Re} p > 0 \\ -ip^{1/2}, & 1/2\pi \leqslant \arg p \leqslant 3/2\pi, \ \operatorname{Re} p < 0 \end{cases} \tag{3.5}$$

$$\beta(p) = \frac{p}{\pi}\int_0^\infty \frac{f^{-1/2}(i\tau)\,\varepsilon(i\tau)}{\tau^2 + p^2}\, d\tau \tag{3.6}$$

In place of (3.6) we have for diagrams *2, 11, 12*

$$\beta(p) = \frac{1}{\pi}\int_0^\infty \frac{f^{-1/2}(i\tau)\,\varepsilon(i\tau)}{\tau^2 + p^2}\, d\tau \tag{3.7}$$

From (3.2), (3.5) - (3.7) we have [1]

$$\operatorname{Re} p > 0, \qquad X(p) = pX^{(*)-1}(p) \tag{3.8}$$

where the superscript (*) means the transposed matrix.

The following asymptotic dependences, valid for large p, result from (3.2) and (3.5) - (3.7):

$$\begin{matrix} \operatorname{Re} p > 0, & X(p) \sim p^{1/2} Q \\ \operatorname{Re} p < 0, & X(p) \sim -ip^{1/2} Q \end{matrix}; \qquad Q = \begin{Vmatrix} \cos q & \sin q \\ -\sin q & \cos q \end{Vmatrix} \tag{3.9}$$

Here the quantity q is given for all diagrams except *2, 11, 12* by the formula

$$q = \frac{\sin\theta_1}{\pi}\int_0^\infty f^{-1/2}(i\tau)\,\varepsilon(i\tau)\, d\tau \tag{3.10}$$

and for diagrams *2, 11, 12* by

$$q=\frac{\sin^2\theta_1}{\pi}\int_0^\infty i\tau f^{-1/2}(i\tau)\,\varepsilon(i\tau)\,d\tau \qquad (3.11)$$

In all cases it is verified directly that the quantity q is real.

The solution of the inhomogeneous Hilbert problem (1.12) is given by the formula [2]

$$\varphi(p)=-\frac{X(p)}{2\pi i}\int_L\frac{[X^-(t)]^{-1}\sigma^{\circ+}(t)\,dt}{t-p} \qquad (3.12)$$

Because of the complete identity between (3.8), (3.9) and (3.12) and the corresponding formulas obtained in [1] in the analysis of diagrams *1*, *5*, *6*, let us present the fundamental dependences for the stress intensity vector and the dislocation vector, whose derivation is given in [1].

Designating the stress intensity vector by n two-dimensional vectors with normal and tangential stress intensity coefficients as components, we obtain

$$n=\int_0^1 N(r_0)\,\sigma(r_0)\,dr_0,\qquad -\sqrt{\pi}N(r_0)=QM(r_0) \qquad (3.13)$$

$$M(r_0)=\frac{1}{2\pi i}\int_L r_0^{\,t}\,[X^+(t)]^{-1}G_{jk}(t;\,\theta_1,\theta_2)\,dt \qquad (3.14)$$

Here $N(r_0)$ and $M(r_0)$ are 2×2 matrices, the former of which is the matrix Green's function for the stress intensity vector.

The two-dimensional dislocation vector $v(r)$, whose components are the normal and tangential divergences of the notch edges, is given by the formula

$$v(r)=\int_0^1 V(r,r_0)\,\sigma(r_0)\,dr_0,\qquad V(r,r_0)=V^{(*)}(r_0,r) \qquad (3.15)$$

Here $V(r,r_0)$ is a 2×2 matrix which is the matrix Green's function of mutual displacements of the notch edges and is defined by the relationship

$$\frac{1}{4}EV(r,r_0)=-\int_1^\theta \xi^{-1}N^{(*)}(r\xi)\,N(r_0\xi)\,d\xi=-\int_1^\theta \xi^{-1}M^{(*)}(r\xi)\,M(r_0\xi)\,d\xi \qquad (3.16)$$

Simple approximate formulas can be obtained in each of the fundamental diagrams for the calculation of the matrix function $M(r_0)$, completely analogously to that which has been done in [1] in application to diagrams *1*, *5*, *6*.

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